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Shop Class for the Next Millennium: Education Through Computer-Enriched Handicrafts

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Abstract: In this paper we use our experiences with the HyperGami program as a springboard for a broader look at the future of computationally-enriched handicrafts. HyperGami is an educational application for the design and construction of mathematical models and sculptures in paper; as such, it serves as a source of examples and insights for the more general problem of how to integrate the "high-tech" features of computation with the "low-tech" features of traditional craft materials in education. We begin by describing the HyperGami program, focusing on those features that were designed in response to problems encountered by papercrafters; we illustrate the program's capabilities by presenting some of our own and our students' papercraft designs; and we describe our initial steps in implementing elements of HyperGami on the World Wide Web. In the closing sections of the paper, we explore the broader educational issues involved in integrating computation and handicrafts; and we conclude with a discussion of how physical objects could play a role in a future "educational object economy."

Keywords: Computational crafts, HyperGami, mathematical papercrafts, educational object economy.

Demonstrations: The authors' *HyperGami* website includes Java applets for rotating and constructing polyhedra <http://www.cs.colorado.edu/~eisenbea/hypergami>, plus two QuickTime videos of an animated *HyperGami* sculpture, included with this article. *HyperGami* is available free of charge, and runs on any color Macintosh with at least 12 MB of free RAM. Email nishioka@cs.colorado.edu for further information. A Java version called *JavaGami* will be released soon. Subscribe to the article's commentaries area to receive this announcement.

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Figure 1. *A paper Venus flytrap*

1. Introduction

Sitting on our desk is a paper sculpture of a Venus flytrap—actually, it looks rather more like the carnivorous plant in the musical *Little Shop of Horrors* than anything else. (See Figure 1 for a portrait.) The pot in which the plant sits is a truncated octagonal pyramid decorated with computer-created geometric figures; the stem of the plant is a tall, thin hexagonal prism; the leaves are also prisms, these squat and oval-shaped, capped to produce a slight bulge; the plant's green "head" is made of two halves of a stretched icosidodecahedron; the evil-looking jaws are lined with tetrahedral teeth. Our next step—to be taken over the coming weeks—will be to elaborate on this sculpture in several ways: a strain sensor will be placed in the lower jaw of the plant, positioned so as to read higher values of bending strain when the jaw of the plant is slightly depressed. Running through the stem of the plant will be a length of Nitinol "muscle wire" (Gilbertson, 1993), a material that shortens by about 5 percent when a small current is placed across it. Both the sensor and Flexinol will be attached to a small programmable Lego brick—the so-called "cricket"—prototypes of which are currently in development at MIT's Media Lab. (Cf. Resnick, 1993) Ultimately, when the jaw of the plant is jostled, the strain sensor should send a signal to the cricket, which can be programmed to send repeating pulses through the Flexinol wire, pulling on the upper jaw of the plant and causing the sculpture to "chew" slowly over the next several minutes.

To tell the truth, we're not unshakably certain that the planned sculpture will work as designed, though it certainly sounds feasible. In any event, we hope that most visitors to the office will find the sculpture humorous; the uncharitable will regard it as merely silly. But the proposed carnivorous plant does in fact represent something beyond an office novelty. The paper forms of the sculpture are designed by a computer program; indeed, the sculpture itself can be regarded as a running computer program, albeit a simple one, embodied in a paper shell. Neither crickets nor muscle wire existed a generation ago; the one is still experimental but not wildly expensive, and the other is marketed commercially at a cost affordable to craftspeople (though perhaps still a bit expensive for classrooms). The strain sensor, while noisy in its response, is made of inexpensive lightweight plastic. Even the color printer on which the sculpture's pieces are printed is far cheaper now than it was a decade ago. In short, the artwork as conceived—in its materials, in its construction, in its cost—is just a bit beyond the current means of schoolchildren and garage tinkerers; it requires only mild optimism to predict that its components will, in the coming decades, be viewed as craft materials—the next century's version of paper and clay and wax and string.

This paper is devoted to the future of educational crafts; it is both a speculative look at how emerging technologies (including the World Wide Web) may affect what the next century's children will do with their hands, and a progress report on our own exploratory efforts in this area. The paper may also be read as a response—a counterweight—to the rhetoric of "virtuality" that pervades much writing on educational computing, and on computational media in general. While many writers look hopefully toward an increasingly "non-physical" future—a future in which people are no longer encumbered by the supposedly bothersome limitations of time, space, the body, and physical materials—we prefer the notion of a future in which the material world itself is a richer place, filled with new stuff for our collective hands and minds to play with.

For the most part, our ideas on this theme have been shaped by our experience in creating and working with the *HyperGami* program—an educational CAD system for the creation of paper polyhedral models and sculptures. (This is the system with which the aforementioned Venus flytrap was created). In the following section we briefly describe the current version of HyperGami, and discuss the program as a tool for "educational craft." The third section continues this theme with a description of more recent work in making HyperGami-inspired tools available on the World Wide Web. In the fourth section we use our experiences with HyperGami as a springboard for discussing what we view as important themes in integrating computational media with handicrafts, particularly in educational settings; and we discuss what other researchers have had to say on these issues. The fifth section is frankly speculative in nature, as we imagine how physical objects could play an important role in the future "educational economy of objects" argued for by Spohrer, Sumner and Buckingham Shum (1998, this issue).

2. HyperGami: Integrating Computational Media and Geometric Papercrafts

2.1 The HyperGami System: an Overview

The HyperGami system was written by the authors and runs on all Apple Macintosh computers with at least 16M of memory. The system has been described in some detail elsewhere (Eisenberg and Nishioka, 1997a; Eisenberg and Nishioka, 1997b), so here we present only an outline of HyperGami for the purposes of motivating the discussion in the remainder of the paper.

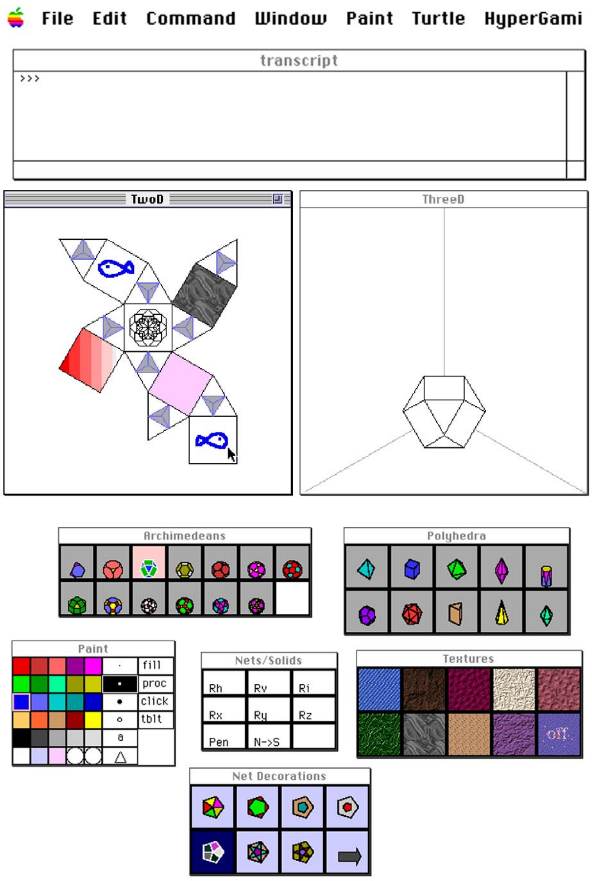

Figure 2. *The HyperGami screen in the course of a sample project (decorating the net of a cuboctahedron). The windows are described in the accompanying text.*

Figure 2 depicts the HyperGami screen in the course of a sample project. The figure shows a variety of windows on the screen. The two most prominent windows are labelled "ThreeD" and "TwoD"; these are the windows in which a newly-constructed solid will be shown in a three-dimensional rendering and in its "unfolded" view, respectively. In Figure 2, the user is in the process of creating a model of a cuboctahedron, one of the thirteen Archimedean solids [1] ; this shape has been presented in the ThreeD window, and is shown as a *folding net* pattern in the TwoD window. The Polyhedra window, and the nearby Archimedeans window, present palettes of starting shapes from which the user may select; these shapes are regarded as "primitives" in the HyperGami environment. Thus, for the Figure 2 scenario, the user began by selecting the cuboctahedron from the Archimedeans window.

There is still more to say about Figure 2; having selected the cuboctahedron, the user is now in the process of decorating the faces of the shape. There are numerous ways of decorating a HyperGami folding net—some of these ways are available by means of the program's direct manipulation interface, and other, more complex techniques are available through HyperGami's built-in Scheme interpreter. The transcript window (at upper left) is the interface to the system's "enriched" version of the Scheme language; HyperGami is built in MacScheme [2] , and both incorporates and extends the basic language system provided by MacScheme. Thus, the transcript window is one provided in the underlying MacScheme system; but the language itself has been extensively augmented with vocabulary specific to HyperGami, and related to the tasks of designing solids and decorating folding nets.

In the figure, the user has decorated a couple of the square faces "by hand", using the mouse; one square is filled with a color texture (selected from the Textures window shown); another with a color gradient; another with a geometric pattern created by a program written to control a Logo-style turtle (Abelson and diSessa, 1980). The triangular faces have been decorated with geometric patterns selected from the Net Decorations window also shown in the figure. When one of the icons in this Net Decorations window is selected, an illustrative Scheme expression is presented in the transcript window; by evaluating this expression, the user is able to decorate faces of the folding net in the manner depicted by the selected icon.

The techniques illustrated by Figure 2 do not begin to exhaust the capabilities of the HyperGami system. There are several additional optional windows not shown in the figure—a palette of still more starting shapes (the thirteen "duals" to the Archimedean solids); a window

1 *The thirteen Archimedean solids (Holden, 1971) are an important class of semiregular solids—i.e., polyhedra all of whose faces are regular polygons and all of whose vertices are surrounded by identical arrangements of faces. In the case of the cuboctahedron, the shape is composed of six squares and eight equilateral triangles.*

2 *MacScheme. Lightship Software. Available through Academic Distributing, Dewey AZ*

through which the user may decorate nets with paint-program-style patterns; and a "Solid Operations" window, analogous to the "Net Decorations" window shown, through which the user may customize HyperGami solids. This last window is shown in Figure 3, and is sufficiently important to merit additional discussion.

By selecting one of the dozen available choices in the Solid Operations window [3] , the user may apply a variety of alterations to HyperGami solids, thereby creating new polyhedra. Selecting a solid operation in fact results in the display of a relevant Scheme expression, much as in the case of the "Net Decorations" window; by evaluating the newly-printed expression, the user may create a new solid as a variant of the current solid. Figure 3 depicts an example of this idea. Here, the user has elected to "cap" one of the faces of the cuboctahedron, resulting in a shape with a spire at the top. Once created, the user has typed a Scheme expression to "unfold" the shape; in Figure 3, we see both the new solid shape and its associated folding net.

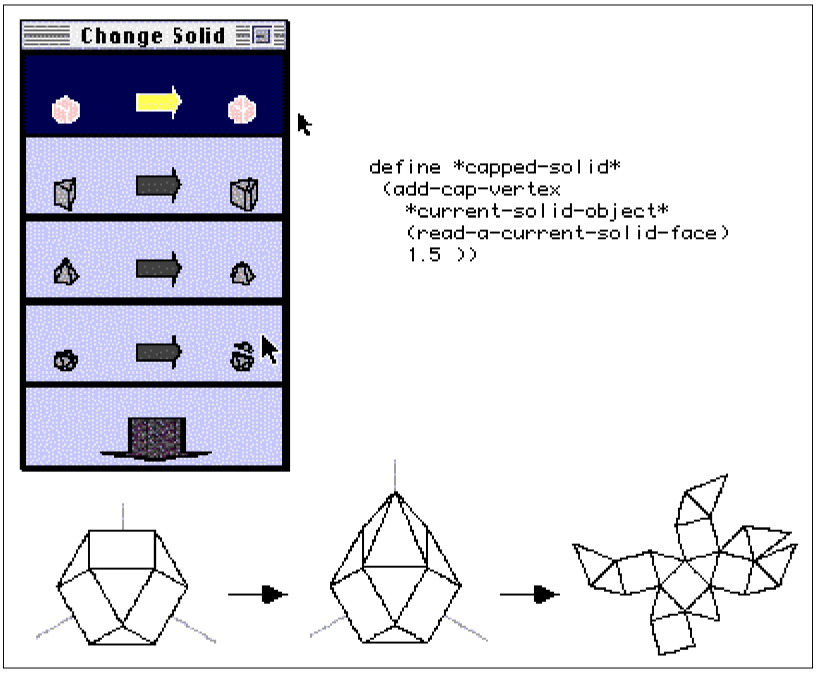

Figure 3. *Customizing (and unfolding) a new HyperGami solid. At top, by selecting an icon in the Change Solid window, we are provided with a Scheme expression that we can edit (right top). At bottom, we use the customization operation to add a new "cap vertex" to the top face of the cuboctahedron; the new shape is unfolded to produce the net at bottom right.*

[3] *Only four operations are visible at one time, but the user may change the visible set by selecting the "arrow" icon toward the bottom of the Solid Operations window.*

To summarize the discussion of the HyperGami system thus far, Figures 2 and 3 represent the fundamental activities of the HyperGami user: creating customized polyhedra, "unfolding" those shapes into folding nets, and decorating the nets, with the ultimate intent of creating novel polyhedral models and sculptures like the plant in Figure 1. Much more could be said about the numerous operations available to the HyperGami user, about the features present in the system, and about our own and our students' experiences in employing the program (Eisenberg and Nishioka, 1997a, and Eisenberg and Nishioka, 1997b include more detail on all these matters). For our current purposes, however, the crucial point is that HyperGami is a system built for the purposes of what might be called *educational crafting*—i.e., a style of craft work associated with rich content (in this case, mathematical content). The remainder of this section expands on these craft-related aspects of HyperGami, and the sections to follow discuss the larger questions of designing software applications appropriate for educational crafting.

2.2 HyperGami as a Medium for Craft

The previous paragraphs provided a broad outline of HyperGami's purpose and operation. In this section, we focus on those aspects of the application that bring into especially high relief its relationship with craft activities. In every case, these features were not planned in the initial design of the system; rather, they arose from the physical activity and materials themselves—and from observations of difficulties that we and our students had in creating polyhedral models. Thus, these features may be viewed as conscious attempts to make our computational environment more responsive to the specific needs of the practicing paper modeller.

2.2.1 Linked Nets and Solids

Perhaps the central problem in decorating a polyhedron's folding net—and a problem that students run into almost immediately in their work—is the task of imagining what a particular decoration, in its "flat" form, will look like when the folding net is finally assembled into a three-dimensional solid. To take a simple case, we might wish to create an octahedron decorated so that pairs of opposite faces are in the same color. Only one of the folding nets in Figure 4 will in fact achieve this aim; we invite the reader to decide which is the correct net.

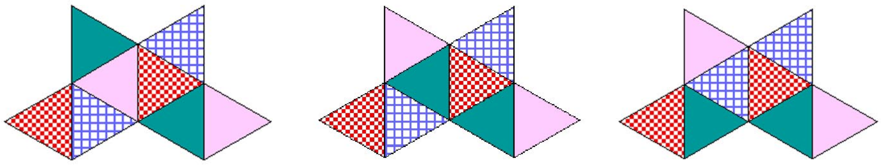

Figure 4. *Three attempts to decorate an octahedron so that each pair of opposite faces has a characteristic shading.*

HyperGami includes two features intended to render this sort of judgment easier for the user. The first allows the user to draw with the mouse on the folding net; as she does so, the system simultaneously presents a line in the appropriate position on the visible 3D solid on the screen. Figure 5 shows this feature in operation: the user is drawing a line on the folding net of the octahedron, and the system is displaying not only that line but also the line that would appear if the folding net were to be assembled into a 3D solid. In HyperGami, this "pen mode" is selected via the Nets/Solids window (visible in Figure 2); and in this mode, if the user drags the mouse over a region off the net (or over a face of the net not visible in the 3D window), the pen simply does not draw at all.

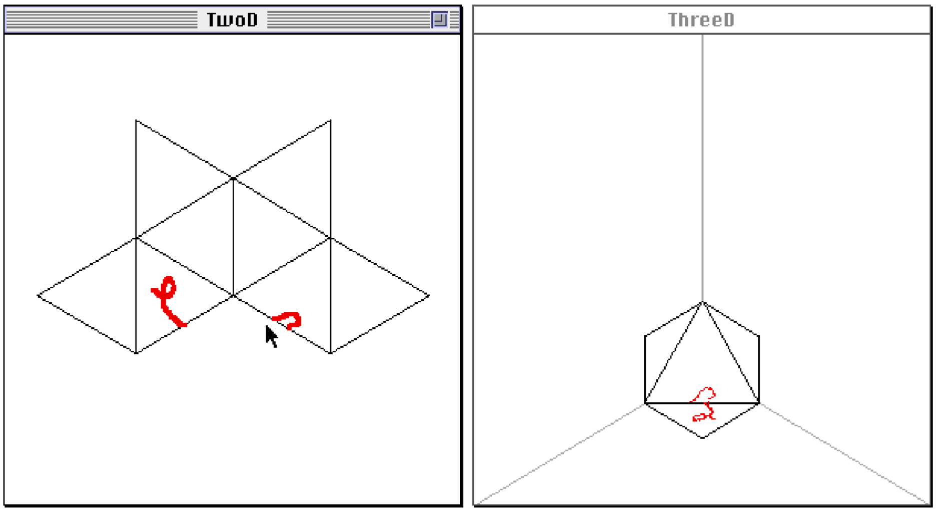

Figure 5. *HyperGami's "linked net and solid mode" in operation. As the user draws a line on the folding net in the TwoD window at left, the system simultaneously presents the line that would appear on the folded solid in the ThreeD window at right.*

This feature allows the user to get an initial feeling for how the individual faces of the folding net on the screen correspond to their 3D realization in the assembled model. By drawing a line that leaves the boundary of one face and moves onto another (as in Figure 5), the user can for instance observe how two boundary edges of the net will join together in the final solid. A second, more elaborate (and much slower) tool allows the user to decorate an entire net and then "paint" that decoration over the visible solid in HyperGami's 3D window. Figure 6 shows this feature in operation: the user has first decorated the octahedral folding net shown at left and then has directed the system to show her what the eventual solid will look like. (This feature is likewise activated by a selection from the Nets/Solids window.) The 3D rendering of the octahedron now shows how the solid will appear, from this particular angle, when the model is created.

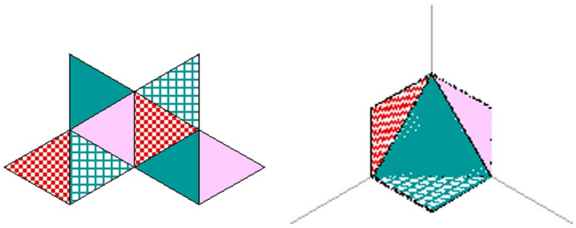

Figure 6. *The shading of a decorated net (left) is transferred to the 3D rendering of the solid (at right).*

2.2.2 Net Customization

In our early work with HyperGami, we often ran into situations in which the program provided us with a folding net that was mathematically correct—i.e., a technically correct unfolding of the desired solid—but otherwise disastrous. Figure 7 shows an example. Here, we are trying to create an approximation to a cone—a pyramid on a regular octagonal base. HyperGami provides us with a folding net that will, indeed, produce a pyramid; but typically, no paper crafter would come up with a net of this sort, since it is fiendishly hard to join together those eight tall triangles into a single vertex. In fact, this is an illustrative example of a more general idea—the difficulty of formalizing, in purely mathematical terms, what it means to produce a "realistic" (and not merely technically correct) solution to an algorithmic problem derived from human practice. (Eisenberg, 1996)

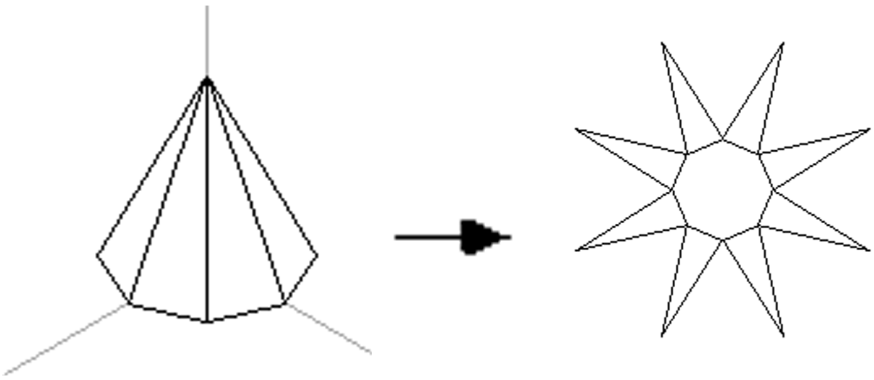

Figure 7. *A pyramid on an octagonal base is unfolded into an especially difficult net.*

In response to this problem, HyperGami includes several features whose purpose is to give the user greater flexibility and control over the attributes of folding nets. For instance, the user can specify (via a menu choice) that the unfolding algorithm employed by the program should begin by placing a face with the fewest possible vertices (among other possible choices); this results in the folding net shown at left in Figure 8. Yet another menu selection permits the user to specify (among other choices) that the unfolding algorithm should work by adding, whenever possible, to a face with fewer vertices; in this particular case, the choice would effectively mean that the unfolding algorithm will progressively place new faces on the edges of already-placed triangles. The resulting net is shown at right in Figure 8, and is much closer to the sort of net that a typical crafter would want to work with.

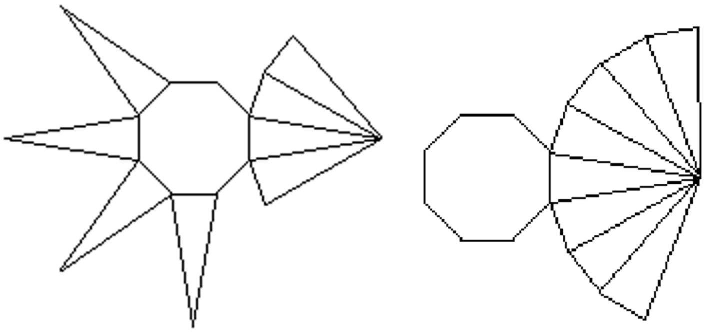

Figure 8. *Two alternative nets for the pyramid shown in Figure 7, produced by varying parameters of HyperGami's unfolding algorithm.*

Still another technique that the experienced HyperGami user can employ is to customize a candidate folding net produced by the system. This technique (which we have dubbed "net twiddling") allows the user to "slide" faces from one location on the net to another, as long as the face is connected to the rest of the net by only one edge. Figure 9 shows the idea in operation, starting with the net at right in Figure 8. Here, we wish to move the octagonal base of the pyramid to a location at the end of the set of triangles. To accomplish this, we use HyperGami's Scheme language to invoke the net-customization operation:

```
(twiddle-current-net)
```

The program now prompts us to select the polygon that we wish to move; once that face is selected, the program "flashes" the candidate edges to which we could move the given face; and finally, once we select the face to which we want to attach the moved polygon, the program recalculates the appropriately customized folding net.

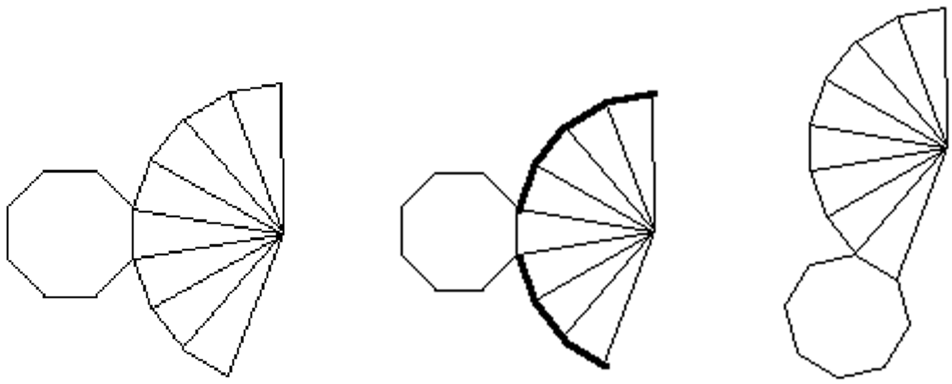

Figure 9. *Customizing a folding net. By selecting the octagon in the original net at left, the system highlights those edges to which the selected face may be reattached (center). At right, we have chosen to move the octagon to the bottom triangle.*

2.2.3 Computing Tabs

Constructing a HyperGami-generated model typically involves cutting out a folding net from paper, creasing edges, and taping or gluing matching edges together. For younger children, taping solids along the outside of the edges is the easier path; but the resulting models are less appealing. Older and more experienced modellers instead add extra paper "tabs" to some of the folding net's outer edges; these tabs are then coated with glue and folded under the matching edge to produce a seamless connection between two faces.

Until very recently, HyperGami did not itself add visible tabs to the folding nets; rather, users would simply cut extra paper regions around a folding net before assembling the model. [4] This resulted in some additional effort for the user, but not so much as to be truly objectionable. In the past several months, however, we have begun creating (and marketing) stand-alone HyperGami "kits" of pre-made pieces for assembly; and for these purposes, asking the customer

4 *To be specific—and for the knowledgeable reader interested in detail—the usual technique would be to add extra tabs to every edge when the entire net was first cut from the paper; as the net was assembled, the user would subsequently cut away any unnecessary tabs. One reader of an earlier draft asked whether students might, in fact, benefit from the process of calculating tabs. The honest answer to this is that we simply don't know—such a task might in fact provide good exercise in spatial cognition, but it might also prove sufficiently tedious so as to dissuade some students from even attempting complex constructions. In any event, HyperGami's tabbing feature is too new for us to offer, even informally, any empirical observations on this score.*

to cut additional tabs for himself would be seen as an unfair imposition. As a result, we have added a new feature to the system that automatically produces a set of appropriate folding tabs to the nets, as shown in Figure 10.

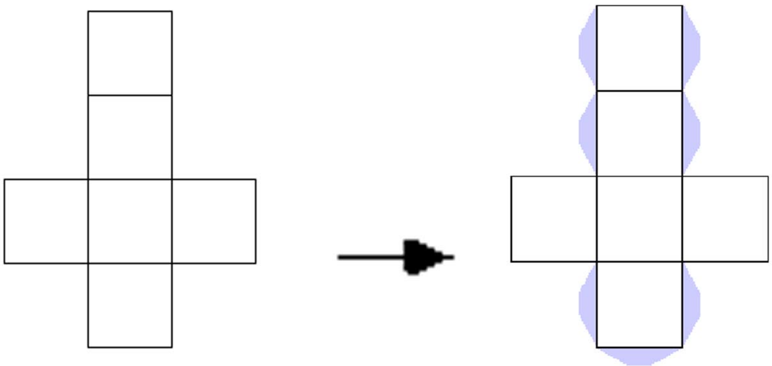

Figure 10. *A HyperGami procedure adds light blue tabs to the net for a cube.*

In summary, then, HyperGami provides a variety of features—linking nets and solids, net customization, tab generation—that derive from the informal, pragmatic needs of the paper modeller. Such features may at first blush seem to respond to low-level, "nuts-and-bolts" problems; but the features might also be viewed as tools that form part of a larger, computationally-enriched craft activity. Taking full advantage of HyperGami's net-customization facilities, for example, becomes something of a skill in itself (one must have a sense of how the system's unfolding algorithm will proceed with a given set of parameters), and designing these facilities involves delicate (and in our own view, still unsatisfactorily resolved) human-computer interface decisions. Computational tools for crafting are entities poised somewhere between the abstract, untouchable world of software objects and the homey constraints of human dexterity; they are therefore creative exercises in making conscious those aspects of craft work (such as the difficulty in folding together the net in Figure 7) that are often more easily represented "in the hands" than in language.

2.3 A Gallery of Computer-Designed PaperCrafts

Before proceeding to a discussion of Web-related work, it is perhaps worth presenting several examples of recent work done in HyperGami, if only to suggest the range of ways in which a single tool may be woven into a variety of craft-related activities. Besides "pure" paper sculptures such as the plant in Figure 1, HyperGami constructions include other materials that are now accessible to the world of color printers: card stock, transparency, and fabric. Figure 11, for instance, depicts a HyperGami iced-tea glass; the glass and ice cubes were printed out on the same sort of acetate sometimes used to make overhead slides for lectures. Figure 12 depicts a "pillowhedron": an icosahedron sewn together from a HyperGami net that was printed onto a paper stock especially designed for transfer to cotton fabric (usually for customized T-shirts).

Figure 11. *An "iced-teahedron" made of acetate transparency and paper.*

Figure 12. *A HyperGami icosahedron realized in fabric.*

Still other craft techniques may be used to decorate polyhedra after assembly. Coating with "imitation copper plating" produces a solid that appears (at least from a distance!) to be made of metal, as in Figure 13; adding mirrors to a newly-created surface produces a "jewel-like" effect, as in Figure 14. Moreover, HyperGami solids may be used as molds for other substances, such as wax, plaster, soap, and chocolate (see Figure 15).

Figure 13. *A small stellated dodecahedron, coated with imitation copper plating.*

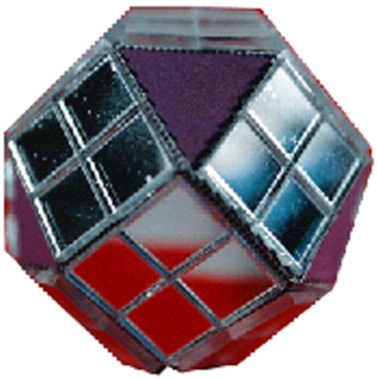

Figure 14. *A cuboctahedron decorated with mirrors on its square faces.*

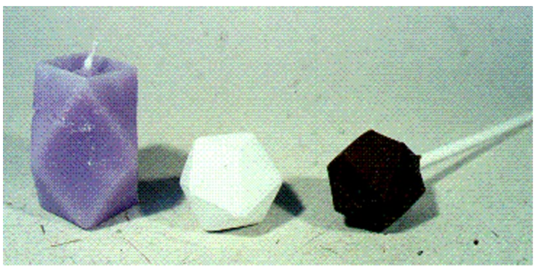

Figure 15. *HyperGami solids in (from left to right) wax, plaster, and chocolate.*

The purpose of presenting these examples is to illustrate the ways in which computationally-enriched craft activities may be integrated with an existing (and still-blossoming) range of materials and techniques. Through these means, the line between "high" and "low" tech educational activities becomes increasingly, and enjoyably, indistinct—a point to which we will return in a later section.

3. HyperGami on the Web: Current and Ongoing Work

Over the past year, we have begun serious efforts at porting elements of the HyperGami system into Java applets available on the World Wide Web. [5] While these efforts are still preliminary, they do suggest ways in which the range of HyperGami activities may be extended beyond that of a single stand-alone application.

The website currently includes two applets written in Java that illustrate at least the potential for a full-fledged Web-based HyperGami. The first, the Platonic Solids applet, is shown in Figure 16. The basic idea behind the applet is that the user can select shapes (via mouse) by pointing to the desired shape as seen in the photograph. Once a shape is selected, the folding net for the shape is presented and the user has a variety of tools with which to decorate it.

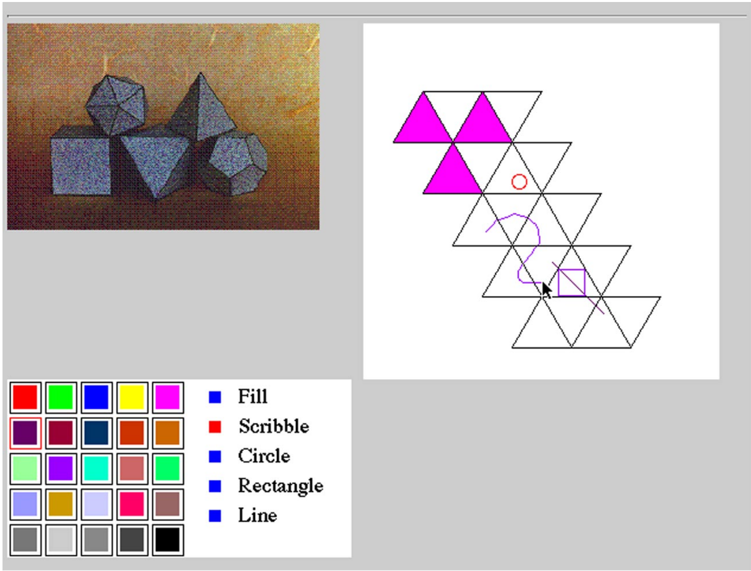

Figure 16. *A view of the Platonic solids applet. Here, the folding net for the icosahedron is being decorated by various means, including a polygon fill routine, rectangle-drawing, circle-drawing, and free-form drawing with the mouse.*

[5] *Our current website can be accessed at <http://www.cs.colorado.edu/~eisenbea/hypergami>*

The Platonic Solids applet is actually a first stab at what we hope will be an interactive "constructionist mathematics book for children" available over the Web. (Nishioka and Eisenberg, 1997) The format for such a book would be similar in spirit to that already visible in the existing applet: a fantasy story would be told through photographs of paper characters, all or most of whom would be constructible (and decorable) via tools that accompany the book. Going only a bit further, such a story book could be extended over time by allowing children to create scenes and figures that could appear in updated online editions of the book. In this fashion, the book could become a much more participatory, and freely evolving, artifact than children's books usually are; our own role would be (in part) that of authors and editors, taking the objects and ideas sent in by children and incorporating the best or most inventive into an ongoing, publicly available story. (The website includes a hint of this idea as well, in that it provides a photographic "sculpture gallery" of HyperGami work done both by us and by students.)

The second applet includes a set of wireframe polyhedra that can be interactively rotated (via mouse) by the user. The viewable polyhedra are the five Platonic solids, along with several customized HyperGami shapes employed in the construction of one of the sculptures (of a penguin) that can also be seen at the website. A view of this applet is shown in Figure 17.

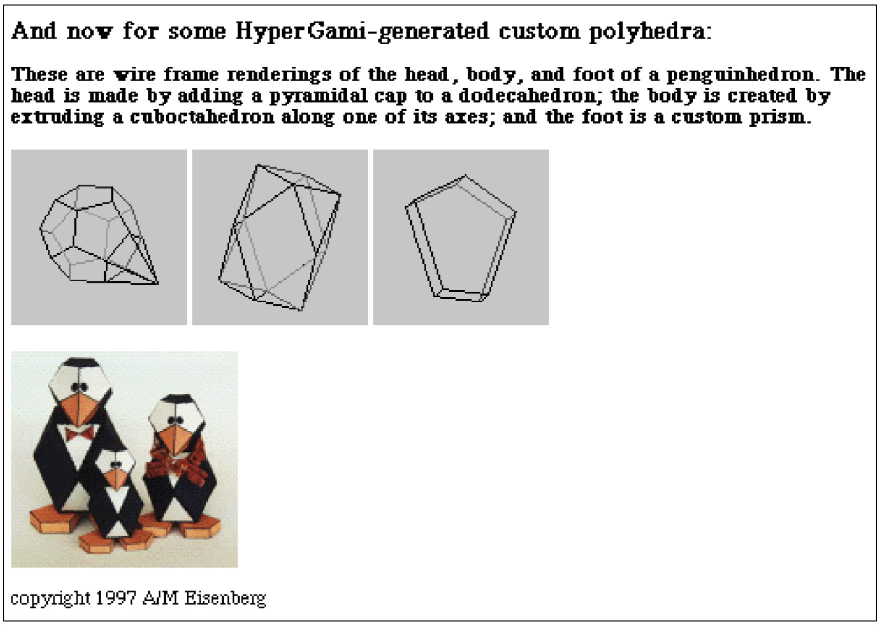

Figure 17. *A view of the "wireframe polyhedra" applet showing the several polyhedral components of a penguin paper sculpture.*

These two applets represent initial steps in what we plan to be a much more extensive porting of HyperGami functionality to the World Wide Web. A large selection of selectable polyhedra, and at least some of the shape-customization facilities of the Scheme program, are currently in preparation as Java programs. Looking somewhat further, the Web also offers the potential for sites in which children can move through environments of interactively manipulable polyhedra—in a sense, such an environment would represent a natural integration of both the "children's book" idea and the second applet in which 3D solids may be manipulated on the screen. In considering these projects, however, we are attempting to tread the delicate line between providing more compelling virtual environments and providing tools that offer compelling reasons to spend time away from the computer screen. In our view, the Web is at its most useful when it is least self-referential—when its purpose is to bring students into creative contact with the world of materials outside the realm of cyberspace. We will return to these issues in the final two sections below.

4. Integrating Computational Media and Crafts

The previous two sections have described current and ongoing work around the HyperGami system itself, focusing on the use of the program as a medium for creative geometric papercrafting. Viewed as a system for creating polyhedral models, HyperGami fits comfortably within a by-now-venerable tradition of geometric modelling in mathematics education—a tradition that goes back at least as far as the eighteenth century, as shown by a set of centuries-old cardboard polyhedral models on display at the University of Göttingen. (Mühlhausen, 1993) In our own century, a large selection of books—often quite beautiful books—present techniques for creating polyhedral models of various forms; our personal favorites include Hilton and Pedersen (1994), Cundy and Rollett (1951), Wenninger (1971), and an astonishing book by Holden (1971). The tradition has recently been extended into computer software as well; current commercial educational systems that incorporate some sort of "folding net" presentation include Tabs+ [6] (which, like HyperGami, also presents examples of using polyhedral parts in larger paper sculptures), and *Shape Up!* [7] (which includes folding nets for the five Platonic solids), while professional CAD programs such as *Touch-3D* 8 and *FormZ* [9] allow solid shapes to be unfolded into nets. To our knowledge, HyperGami is unique in its combination of

- a focus on classical polyhedra and functional variants thereof;
- the use of those polyhedra as the basis of a variety of geometric papercrafting activities

6 *Tabs+. The Knowledge Tree, Inc., 554 Merton Street, Toronto, Ontario, Canada., M4S 1B3.*

7 *Shape Up! Sunburst Communications, Inc. Pleasantville, NY <http://www.sunburst.com>*

8 *Touch-3D. Lundstrom Design, Stockholm, Sweden <http://www.algonet.se/~ludesign>*

9 *Form.Z. Auto.Des.Sys, Inc. Columbus, Ohio.*

(including complex polyhedral modelling, paper sculpture, and the creation of working paper machines);

- an extensive collection of special-purpose computational tools (such as the "net customization" described earlier) motivated by the specific requirements and constraints of paper modelling; and
- the system's design as a programmable application, including both direct manipulation features and a full-fledged interactive programming language suitable for more advanced work.

The point at issue for the moment, however, is not so much what distinguishes HyperGami from related efforts, but rather the larger tradition of mathematical papercrafting of which those efforts form a part. In fact, one might well take a conceptual step back and inquire about the purpose of this tradition. Why have two centuries of mathematicians and educators bothered to construct paper models? And why impose the task on students?

There are several ways of answering such questions. One way focuses on mathematical cognition, and points to research such as that by Piaget and Inhelder (1948), who reported that elementary school children with experience in constructing paper models were approximately two years ahead of those with no such experience in performing tasks involving the visualization of unfolded solids (pp. 275-276). Two decades later, Brinkmann (1966) constructed a curriculum for enhancing spatial thinking that included (among other elements) handling of paper shapes. Indeed, tasks related to paper folding are often used as indicators for measuring spatial thinking generally (Shepard and Feng, 1972; Ekstrom, French, Harman and Dermen, 1976).

Much about this "cognitive" argument on behalf of polyhedral modelling is, admittedly, spotty. It is not clear what precisely about the use of paper models might have accounted for the aforementioned success of some of Piaget and Inhelder's subjects: for instance, is it the construction of these objects, the handling of the objects, or the observation of the objects (or perhaps some combination of these) that makes the crucial difference? Is the use of paper models a particularly efficient means toward teaching the linking of two- and three-dimensional representations of surfaces (or "surface development", as it is termed in the literature), or might there be other, more efficient means as yet unexplored? And—assuming that spatial visualization is an important skill in mathematical thinking (itself a debatable issue)—where does this notion of surface development fit into the broader landscape of visualization skills? Is this a tiny, specialized aspect of spatial visualization, or is it a powerful, central skill? Is teaching surface development as a means toward strengthening spatial cognition analogous to, say, teaching the recognition of palindromes as a means of strengthening spelling skills—not, perhaps, a totally useless effort, but at best a peripheral one? At present, the case on behalf of polyhedral modelling is still incomplete. Still, there is at least a suggestive case to be built from the literature that this activity is worthwhile as part of an effective mathematics education.

There is another, less purely "cognitive" style of answer to the original question, however; and it would be a mistake to assess the value of polyhedral modelling purely in terms of skills such as spatial visualization. The argument, in short, doesn't end here. In fact, once we look at polyhedral modelling in a fresh light—as a craft activity—a variety of features of the activity suddenly appear in high relief:

4.1 Longevity of products

Several paragraphs earlier, we mentioned that there are eighteenth-century polyhedral models still on display at the University of Göttingen. There is—or there ought to be—something startling in this notion: after all, how many other educational artifacts have been thought worthy of survival (much less display) for two centuries? This in itself is worth noticing: polyhedra last. Little else in a student's educational experience—indeed, little else in the professional lives of most people—has the oddly satisfying property of longevity; and certainly the advent of computational media has, for the most part, exacerbated the trend toward making educational efforts more and more ephemeral. A fifth-grader's term paper might survive long enough to be read, with interest and nostalgic pleasure, by its author as an adult; but all the evidence to date indicates that computational media don't survive nearly as well as this. An adult will not be able to view a twenty-year-old computer simulation: not only is it unlikely that the software will be available to run the simulation, it is unlikely that even the hardware will be available. Few programs, processors, or machines can be counted on to last as much as a decade; and the result is that most products of a student's educational life, when realized in computational media, have a disturbingly short history. The issue is discussed eloquently by D. T. Max (1994) in a recent popular article in the *Atlantic Monthly*:

> Ours is a culture that has made a fetish of impermanence. Paperbacks disintegrate, polaroids fade, video images wear out. Perhaps the first novel ever written specifically to be read on a computer and to take advantage of the concept of hypertext... was Rob Swigart's Portal, published in 1986 and designed for the Apple Macintosh, among other computers of its day. The Apple Macintosh was superseded months later by the more sophisticated Macintosh SE, which, according to Swigart, could not run his hypertext novel. Over time people threw out their old computers... and so Portal became for the most part unreadable.... 'It's not clear, with fifty incompatible standards around, what will survive,' says Ted Nelson, the computer pioneer, who has

> grown disenchanted with the forces commercializing the Internet. 'The so-called information age is really the age of information lost.'" (p. 71)

Admittedly, HyperGami polyhedra are probably not as long-lasting as Max might wish—they are unlikely to outlive, e.g., the average paperback book—but compared to the mayfly-like timespan of most computational media in education, HyperGami craft constructions have a reassuring degree of staying power.

4.2 Craft products as objects with personalized meaning

Polyhedral models are, of course, tangible physical objects—but to point this out is not merely to state the obvious. Physical objects are capable of playing interesting roles in people's lives—as souvenirs, as mementoes, as gifts. Repeatedly, in our observations of students using HyperGami, we have been surprised at the ways in which polyhedral models and sculptures take on "social currency" in the lives of their creators. Students have used HyperGami creations as presents, as Christmas ornaments, as objects for display at home; one student created a polyhedral portrait of her pet rooster.

Our observations of HyperGami creations as "personalized objects" is strongly reminiscent of the observations made by Csikszentmihalyi (1993) in his reflections on an interview-based study of people's attitudes toward their household possessions:

> [O]bjects reveal the continuity of the self through time, by providing foci of involvement in the present, mementos and souvenirs of the past, and signposts to future goals.... [O]bjects give concrete evidence of one's place in a social network as symbols (literally, the joining together) of valued relationships. In these... ways things stabilize our sense of who we are; they give a permanent shape to our views of ourselves that otherwise would quickly dissolve in the flux of consciousness. (pp. 22-23)

Perhaps even more striking is the fact that in most writing on mathematics education, little notice is taken of this affective role that objects are capable of playing in students' lives. Few products of a mathematical education are, indeed, designed to take advantage of this role; it is unlikely that a mathematics workbook (even a perfect one!) will be given as a gift, displayed as an artwork, or retained as a keepsake. But, in our view, objects that play such roles may be not only precious personal possessions; they may likewise be educational landmarks. It is plausible, after all, that an object created as a gift may be labored over with an attention beyond that bestowed on an exercise book; it is plausible that a mathematical object displayed at home will be noticed at random occasions, at which time it might be the subject of reverie or contemplation; it is plausible that an object retained as a souvenir may reveal fresh meaning or insight to a student many months after its initial construction. Polyhedral models and their more "artistic" variants are capable of having this sort of additional value.

4.3 Craft objects as exemplars of design

The educational benefit of creating mathematical objects may well go beyond the domain of mathematics itself; such objects are, after all, the results of design projects. The task of building a decorative solid—perhaps more pointedly, the task of visualizing a full-fledged polyhedral sculpture (as in HyperGami) and seeing its construction through—is an exercise in engineering, albeit on a small scale. And engineering is itself an exercise that, in its fullest form, works back and forth between abstract conceptions and tangible materials. As Ferguson writes in his wonderful book *Engineering and the Mind's Eye*:

> Despite its complexity, subtlety, and refusal to fit into neat diagrams... engineering design follows a predictable path whose nature will not be changed by computer assisted design (CAD) or by a wished-for science of design.... To accomplish a design of any considerable complexity—a passenger elevator or a railroad locomotive or a large heat exchanger in an acid plant—requires a continuous stream of calculations, judgments, and compromises that should only be made by engineers experienced in the kind of system being designed. *The 'big' decisions should be based on intimate, firsthand, internalized knowledge of elevators, locomotives, or heat exchangers. Every designer ought to have an intuitive sense of the practical limits of the performance of moving machinery and a broad sense of the adequacy of materials and the fabrication process.* (Ferguson, 1992, pp. 37-38; italics added)

Understanding the relationship between one's ideas and the materials in which they will be realized is an important, usually tacit, aspect of engineering design; and it is an aspect that tends to be undervalued by "pure" computational media, in which design begins and ends in software. A student who works entirely on a computer screen may well be having a valuable educational experience; but it is an experience that is lacking in the unique dimensions that shape the experience of the engineer. To the extent that we view design as an important element in education, we should therefore value projects in which design is carried through from its conceptual beginnings to its tangible end. [10]

10 *This paragraph also highlights the reason why we would use the term "Shop Class" (rather than "Art Class", as suggested by one reader) in our title for this paper. While papercrafting is a discipline that undoubtedly has its artistic side, it is the specifically tangible engineering and "craft" side of the discipline that interests us for the purpose of this paper. In contrast, a focus merely on integrating computation and art would not bring the tangible side of our work into the same sort of high relief: after all, much computational art (whether or not it expressly integrates mathematical ideas) remains as intangible as many other computational products.*

4.4 Craft objects as communicators of ideas

Much research in mathematics education has been devoted to the use of various sorts of concrete materials and "mathematical manipulatives" (Ball. 1992; Resnick and Omanson, 1987; Vochko, 1979; Welchman-Tischler, 1992). Typically, the focus of this research is on the role of concrete materials as mathematical representations: blocks (e.g.) may be used to represent number values, balancing scales may be used to represent multiplication problems, and so forth. While these issues are obviously of tremendous importance, there is another aspect of concrete mathematical objects—particularly when produced through craft activities, as in the case of HyperGami models—that is less frequently noticed and acknowledged: namely, the role of concrete materials as *public* and (consequently) communicative artifacts. The fact that a polyhedral model may be put on display—as the Göttingen models were, and as many of our own students' efforts have been—means that they are not simply private representations, but are instead

representations available to others. Papert (1991) describes this element of public representation as central to his notion of "constructionism":

> Constructionism-—the N word as opposed to the V word—shares constructivism's connotation of learning as 'building knowledge structures' irrespective of the circu stances of the learning. It then adds the idea that this happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe. (p. 1)

Public artifacts have an obvious motivational role: particularly in the domain of mathematics, they permit students to engage in something approximating artistic self-expression. [11] Moreover, such artifacts act as advertisements for the fascination of the domain itself; as Pedersen writes:

> It is surely difficult to believe that anyone who has been exposed to the delights of constructing polyhedra and studying them could as, 'Why should we study polyhedra?' I have these models in my office and students come in and beg to know how to make them. They never ask, 'What are they good for?' They know! And we know too. (Pedersen, 1988, p. 143)

The previous paragraphs have argued that there are valuable aspects of polyhedral models that are consistent with, but not subsumed by, their purely "cognitive" aspects. Polyhedral models and sculptures are objects with staying power; they are objects with potentially strong affective content; they are exemplars of designed artifacts in the material world; and they are potentially

[11] *Cf. the discussion in Csikszentmihalyi, Rathunde and Whalen (1993, p.107 and p.195), in which the motivation of high school students specializing in the arts is contrasted with the relatively lower motivation of students specializing in science.*

expressive, communicative acts. None of these features is, of course, unique to polyhedral models *per se*; rather, they are features that are present in greater or lesser degree in virtually all sorts of craft activities. Similar effects might well be achieved by mathematical crafting in other materials such as yarn or wax (Papert, 1991) describes an effective mathematics class employing soap sculptures, and Laffan (1980) describes his own positive teaching experiences with polyhedral candles). Nor are these effects limited to the mathematics classroom; scientific apparatus, for instance, might well be viewed as "craft objects" in their own right. (See Diehn and Krautwurst, 1994 for some delightful examples along these lines.) Our experiences with students creating HyperGami sculptures have alerted us to these usually tacit aspects of educational craft; but those aspects are, we believe, important in many other settings as well.

There is another important feature of our HyperGami experience, however, that is perhaps more unusual in this context: namely, the integration of computational media and crafts. It would be an error to read the preceding paragraphs either as fundamentally anti-technological or as technologically retrograde. Nothing about the notion of "craft" implies, by necessity, ancient technology; crafted objects may well be designed with computational media [12] , and may even themselves incorporate computational elements. Nonetheless, the reigning computational culture tends to stress precisely those features that are most "un-craftlike". Computational media—particularly in education, and even more particularly in mathematics education—tend to be short-lived, devoid of personalized affective content, intangible in their effects, and private in their use. These observations are especially applicable to question-and-answer mathematical programs, but they are only slightly less applicable to (e.g.) mathematical video games or simulations. Such systems may have great educational value in their own terms, but they will be relatively lacking in the features that make crafting activities such powerful educational experiences.

Much contemporary writing on computational media in fact stresses and celebrates the most intangible, nonphysical aspects of virtual environments and "cyberspace". Rheingold (1990), in an early paper on the subject, wrote "Cyberspace is a human-computer interface, but it is also a mind-space, the way mathematics and music and myth are mind-spaces—mind-space you can walk around in and grab by the handles." (p. 449) Negroponte, in his enjoyable book *Being Digital*, describes the cultural contrast between cutting-edge computational media and the physical world:

> In the same ways that hypertext removes the limitations of the printed page, the post information age will remove the limitations of geography. Digital living will include less and less dependence upon being in a specific place at a specific time, and the

12 *An interesting example along these lines is provided by the Barbie Fashion Designer software (Mattel Toys, El Segundo, CA), through which children may decorate and print out (directly onto fabric) fashion designs for dolls.*

> transmission of place itself will start to become possible. If I really could look out the electronic window of my living room in Boston and see the Alps, hear the cowbells, and smell the (digital) manure in summer, in a way I am very much in Switzerland. (Negroponte, 1995, p. 165)

Benedikt (1991) describes a hypothetical cyberspace in even more poetic terms:

> Cyberspace: A new universe, a parallel universe created and sustained by the world's computers and communication lines. A world in which the global traffic of knowledge, secrets, measurements, indicators, entertainments, and alter-human agency takes on form: sights, sounds, presences never seen on the surface of the earth blossoming in a vast electronic night. (p.1)

Such visions are powerful and compelling; and quite probably, they accurately describe an aspect of computational life that will take on ever-greater importance in the near future. But a relentless and exaggerated focus on "virtuality" is, in our view, myopic. The lived-in day-to-day physical world is itself an endlessly rich space, filled with fun objects whose subtler aspects—longevity, serendipity, nostalgia—will inevitably be captured only imperfectly in less tangible media. The traditional physical materials of craftwork—paper, wax, string, clay, fabric, wood—present challenges and promise rewards that pure graphics and information inevitably fail to capture. Beyond those general features of craft materials mentioned earlier, each specific material offers its own tactile approach to a particular type of mathematical or physical abstraction: paper becomes an approximation to a 2D surface of fixed area; string an approximation to a conduit of mechanical force; wood dowels to rigid noncompressible beams; and so forth.

Moreover, craft materials do not end at those substances that, in the present day, we have come to regard as "low tech." The past half-century has witnessed an explosion of new, relatively inexpensive materials whose potential for educational crafting has yet to be fully explored. Styrofoam, plasticine, mylar, and Velcro all offer new constraints and possibilities to the educational craft designer; lenses, color filters, fiber optics cables, and diffraction gratings are now available at low cost; temperature-sensitive materials and paints are marketed in science catalogs; materials with "memory", such as the aforementioned Flexinol wire, are slowly finding a place in home and school workshops. These materials might well be dubbed "middle tech" objects: they are products of recent technological innovation, but may nonetheless be regarded simply as interesting, unforbidding stuff.

Building on our experiences with HyperGami, then, we may now envision a continuation of the general idea of integrating computers and crafts, but with an eye toward incorporating an ever richer range of materials. In a sense, the advent of "middle tech" materials might be seen as an opportunity for elaboration or enrichment of HyperGami on the physical or tangible side, by expanding the notion of educational craftwork. [13] On the other hand, we might also envision an elaboration of HyperGami on the computational side, integrating craft objects with ever-more-powerful information technology. This is the subject of the final section below.

5. Craft Objects in an Educational Object Economy

The previous discussion alluded to the burgeoning world of craft materials appropriate to education. Somewhat more speculatively, that world of materials might become increasingly integrated with computational media—i.e., the lines between "low", "middle", and "high" tech could well become thoroughly blurred. Fabrics might have computational elements woven into them (a trend already visible in the move toward "wearable computers" (Mann, 1997); paper constructions could be integrated with low-cost computational elements (a trend already visible in the appearance of electronically augmented greeting cards and magazine ads); the standard equipment of woodworking—nails, screws, hinges—could be augmented with (say) "programmable" hinges and joints that could be used to make dynamic wooden structures of novel kinds.

One needn't regard this phenomenon from only one vantage point, as the incursion of computational elements into the realm of craft materials. Conceivably, the incursion works both ways. Craft objects are, after all, objects; and while classically, the "objects" of software engineering are pure software, we might allow our imaginations to encompass a style of object-oriented programming that spans a wide spectrum of physical realization. In this sense, the "educational object economy" described by Spohrer, et al. (1998, this issue) might eventually consist of repositories of software objects augmented and enriched by craft objects of various sorts. Several possible examples of the "multi-object economy" are suggested below.

5.1 Objects with Websites

In a world in which almost every advertisement is accompanied with a WWW address, it is hardly a stretch to imagine that objects themselves might routinely be annotated with the addresses of "object-specific home pages." In the realm of craft, such annotation could play several possible roles. A craft construction (such as the result of a woodworking project) could have a website explaining how the object was built, complete with explanatory video; a new craft material (such as temperature-sensitive paint) could have a website through which craft practitioners may submit examples of creative ways in which the material was used; a novel tool could be accompanied by Web documentation explaining its use. In these ways, the proliferation of websites could be employed to make materials more self-explaining and accessible; thus, rather than distancing people from the physical world, the Web might come to be actively employed

13 *Eisenberg and Eisenberg (1998) is devoted to a much more extended discussion of these issues.*

to promote a "craft culture" in which physical materials and objects are seen as opportunities for educational play rather than as mysterious "black boxes" manufactured by unknown means and employed only by experts.

5.2 Programmable Craft Objects with Downloadable Behavior

Certain types of programmable craft materials—the programmable brick is an early example—might be capable of an extremely wide range of complex behaviors. In this case, one might well wish to browse an online repository of potential behaviors or construction possibilities involving that object; and the software elements of such complex constructions could be downloadable in much the same fashion as any other program. For instance, a programmable homemade marionette consisting of (among other elements) several embedded computers could have the default behaviors of its limbs altered by downloading new programs into those components.

5.3 Craft Advisors

One of the recurring themes in the literature of scientific computation is the quest toward automated intelligent assistance (cf. Abelson et al., 1989; Zhao, 1974; Bradley, 1992). Broadly speaking, the purpose of this research is to design computational systems that can assist the working scientist by (e.g.) suggesting simulation parameters, predicting and interpreting the results of computational experiments, and offering intelligent help with choices of numerical methods. It is fair to say that, as a whole, the literature of "intelligent scientific computation" has focused exclusively on the intangible, purely informational aspects of scientific practice: simulation, data visualization, and so forth. But for many scientists (and students of science), there is a strong craft element to their work: a scientist might wish to know how to prepare a solution with certain properties, how to produce glassware of a certain shape, how to find a particular adhesive, how to produce a mechanical linkage with a given geometry, or how to rig up certain types of optical apparatus. Such craft elements of scientific practice were perhaps more apparent in the days of aristocratic practitioners such as Lavoisier and Boyle; but again, with the proliferation of intermediate tech materials, scientific practice might well shift back in the direction of low cost, personalized construction of special-purpose scientific instruments (ideally in a much more democratic fashion than in earlier centuries). A related development on the computational side, then, would be the extension of "intelligent assistance" to this crafting aspect of science: an advanced environment for computational science might well advise the scientist (or student) not only in which integration method to use for a particular simulation, but also in such matters as how to build (e.g.) a relatively inexpensive pressure regulator with a particular geometry.

Notions such as these—objects with web addresses, programmable craft objects, and intelligent advisors for craftwork—suggest an educational future which in some respects runs counter to the reigning aesthetic of virtuality. Craft activities demand patience, a long attention span, and above all, ample free time of their practitioners; they are thus ill-suited to users who expect the pacing of a video game. Craft activities celebrate mess and material constraints, even as they offer a burgeoning range of new materials; they are thus ill-suited to "virtual laboratories" in which objects are resolutely untouchable and imaginary. Craft activities produce objects whose value derives from long-term personal memory; they are thus ill-suited to worlds in which value primarily derives from expense and novelty. And craft activities, particularly as practiced by children, are capable of investing real objects with the designer's personality; they thus run counter to a culture of Web-based education that, in our view, threatens to lead its charges into adulthood without a single souvenir.

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References

Abelson, H., Eisenberg, M., Halfant, M., Katzenelson, J., Sacks, E., Sussman, G.J., Wisdom, J. and Yip, K. (1989) *Intelligence in Scientific Computation*. Communications of the ACM, 32 (5), 546-562.

Abelson, H. and diSessa, A. (1980) *Turtle Geometry*. Cambridge MA: MIT Press.

Ball, D.L. (1992) *Magical Hopes: Manipulatives and the Reform of Math Education*. American Educator, (Summer), 15-18; 46-47.

Benedikt, M. (1991) Introduction to M. Benedikt, (Ed.) *Cyberspace: First Steps*. MIT Press, Cambrdige, MA, pp. 1-25.

Bradley, E. (1992) *Taming Chaotic Circuits*. (Ph.D. Thesis) MIT Lab AI Technical Report 1388.

Brinkmann, E. (1966) *Programmed Instruction as a Technique for Improving Spatial Visualization*. Journal of Applied Psychology, 50 (2), 179-184.

Csikszentmihalyi, M. (1993) *Why We Need Things*. In S. Lubar and W. D. Kingery (Eds.), History from Things. Washington, DC: Smithsonian Institution Press.

Csikszentmihalyi, M., Rathunde, K. and Whalen, S. (1993). *Talented Teenagers*. Cambridge, UK: Cambridge University Press.

Cundy, H. M. and Rollett, A. P. (1951) *Mathematical Models*. London, Oxford University Press

Diehn, G. and Krautwurst, T. (1994) *Science Crafts for Kids*. New York: Sterling.

Eisenberg, M. (1996) *The Thin Glass Line: Designing Interfaces to Algorithms*. Proceedings of CHI96: Conference on Human Factors in Computing, Vancouver, April, 1996, 181-188, ACM: NY.

Eisenberg, M. and Nishioka, A. (1997a) *Orihedra: Mathematical Sculptures in Paper*. To appear in International Journal of Computers for Mathematical Learning.

Eisenberg, M. and Nishioka, A. (1997b) *Creating Polyhedral Models by Computer.* To appear in the Journal of Computers in Mathematics and Science Teaching.

Eisenberg, M. and Eisenberg, A.N. (1998) *Middle Tech: Blurring the Division between High and Low Tech in Education*. To appear in A. Druin, (Ed.) The Design of Children's Technology. San Francisco: Morgan-Kaufmann.

Ekstrom, R., French, J. and Harman, H. with Dermen, D. (1976) *Manual for Kit of Factor Referenced Cognitive Tests*. Lawrenceville, NJ: Educational Testing Service.

Ferguson, E. (1992) *Engineering and the Mind's Eye*. Cambridge, MA. MIT Press.

Gilbertson, R. (1993) *Muscle Wires Project Book*. Mondo-Tronics.

Hilton, P. and Pedersen, J. (1994) *Build Your Own Polyhedra.* Menlo Park, CA: Addison-Wesley.

Holden, A. (1971) *Shapes, Space, and Symmetry.* New York: Dover.

Laffan, A. J. (1980) *Polyhedron Candles: Mathematics and Craft*. Arithmetic Teacher, Nov., 18-19.

Mann, S. (1997) *Wearable Computing: a First Step Toward Personal Imaging*. IEEE Computer, 30 (2), 25-32.

Max, D. T. (1994) *The End of the Book?* Atlantic Monthly, 274 (3), 61-71.

Mühlhausen, E. (1993) *Riemann Surface—Crocheted in Four Colors. Mathematical Intelligencer,* 15 (3), 49-53.

Negroponte, N. (1995) *Being Digital.* Vintage Books, New York.

Nishioka, A. and M. Eisenberg, (1997) *Paper Modelling from a Distance: Computational Crafts on the Web*. Proceedings of AACE ED-MEDIA/ED-TELECOM 97, Calgary, August, 1997, pp. 757-762.

Papert, S. (1991) *Situating Constructionism.* In Harel, I. and Papert, S. (Eds.), Constructionism. Norwood, NJ: Ablex.

Pedersen, J. (1988) *Why Study Polyhedra?* In M. Senechal and G. Fleck (Eds.), Shaping Space: A Polyhedral Approach, pp. 133-147. Boston: Birkhäuser.

Piaget, J. and Inhelder, B. (1948) *The Child's Conception of Space*. New York: W. W. Norton & Company.

Resnick, L.B. and Omanson, S.F. (1987). *Learning to Understand Arithmetic*. In R. Glaser (Ed.), Advances in Instructional Psychology, (Vol. 3, pp. 41-96). Hillsdale, NJ: Erlbaum.

Resnick, M. (1993) *Behavior Construction Kits*. Communications of the ACM, 36 (7), 64-71.

Rheingold, H. (1990) *What's the Big Deal about Cyberspace?* In B. Laurel, (Ed.), The Art of Human-Computer Interface Design, pp. 449-453. Reading, MA: Addison-Wesley.

Shepard, R. and Feng, C. A (1972) *Chronometric Study of Mental Paper Folding.* Reprinted in R. Shepard and L. Cooper (Eds), Mental Images and Their Transformations, pp. 191-206. Cambridge, MA: MIT Press.

Spohrer, J., Sumner, T., and Buckingham Shum, S. (1998, this issue) *Educational Authoring Tools and the Educational Object Economy: Introduction to this Special Issue.* Journal of Interactive Media in Education, 98 (10) <http://www-jime.open.ac.uk/98/10>

Vochko, L. (1979) (Ed.) *Manipulative Activities and Games in the Mathematics Classroom*. Washington, D.C.: National Education Association.

Welchman-Tischler, R. (1992) *The Mathematical Toolbox.* White Plains, NY: Cuisenaire.

Wenninger, M. (1971) *Polyhedron Models.* New York: Cambridge University Press.

Zhao, F. (1974) *Extracting and Representing Qualitative Behaviors of Complex Systems in Phase Spaces*. Artificial Intelligence, 69 (1-2), 51-92.